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A Computational Approach to Solving Higher-Order Multi Indexed Fractional Differential Equations in Caputo Sense

Ojobo Solomon Ocheka[1*], Adewale James[2], Apollos Waraba Dibalya[3] & Folake Joseph[4]

[1]Department of Mathematics, School of Pure and Applied Sciences, Modibbo Adama University, Yola, Nigeria. [2]Department of Mathematics and Statistics, School of Art and Sciences, American University of Nigeria, Yola, Nigeria. [3]Department of Mathematics, School of Pure and Applied Sciences, Adamawa State College of Education, Hong, Nigeria. [4]Department of Mathematics, School of Pure and Applied Sciences, Bingham University, Abuja-Keffi Road, New Karu, Nasarawa, Nigeria. Corresponding Author (Ojobo Solomon Ocheka) Email: ojobosolomon4@gmail.com[*]

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ABSTRACT

This paper introduces a novel computational approach for solving higher-order multi-indexed fractional differential equations (FDEs) in the Caputo sense. By leveraging a power series polynomial collocation method, the proposed technique reformulates the FDEs into an equivalent integral form, enabling accurate and stable numerical solutions.

The method addresses key challenges in fractional calculus, including the handling of multi-indexed derivatives and ensuring computational efficiency. Numerical experiments demonstrate the approach's superior accuracy, with error analyses revealing significant improvements over existing methods.

The results underscore the method's applicability to real-world problems in physics, engineering, and biology, where fractional derivatives model memory-dependent phenomena. This work advances the toolkit for computationally solving complex FDEs while maintaining robustness and convergence.

Keywords: Fractional Differential Equations; Caputo Derivative; Multi-Indexed Systems; Power Series Method; Collocation Method; Numerical Approximation; Fractional Calculus; Computational Mathematics; Stability Analysis; Convergence Analysis; Error Estimation.

1. Introduction

Fractional differential equations and fractional integral formulations are essential fundamental instruments across diverse fields such as mathematics, physics, chemistry, and engineering. Frequently emerging within mathematical models of complex real-world phenomena, these equations are often expressed through functional formulations, such as ordinary or partial differential systems, aligning closely with the study of mixed-order fractional differential equations. Unlike classical differential equations, which are limited to integer-order derivatives, fractional differential equations (FDEs) provide a more generalized framework by incorporating derivatives of arbitrary (non-integer) orders. This adaptability enables precise representation of intricate systems that display memory and inherited characteristics, which are frequently found in viscoelastic substances, irregular diffusion patterns, biological mechanisms, and feedback control applications.

In the early 20th century, [20] pioneered the study of integro-differential equations, a novel class of equations designed to model population dynamics. These equations are distinctive due to the presence of one or more derivatives of the target function inside the integral expression. Integro-differential equations have since become pivotal in numerous physical and engineering applications, including the kinetic equations used to describe phenomena such as the dynamics of covering rarefied gas flows, plasma physics, radiation propagation, and coagulative mechanisms [1].

Throughout time, numerous computational methods have been introduced to tackle the challenges involved in solving fractional differential equations. Notable methods documented in the literature include the Perturbed

Collocation Method [9], the Adomian Decomposition Method [13], the Hybrid Linear Multi step Method [12], the Chebyshev-Galerkin Method [12], the Bernoulli Matrix Method [6], the Differential Transform Method [10], the Pseudospectral Method [9], the Bernstein Polynomials Method [11] and the Mellin Trans form Approach [10]. Additionally, some studies propose innovative strategies, such as using an operational matrix based on Boubaker polynomials to approximate solutions for fractional equations involving multiple orders [5]. Another example introduced a technique in addressing Fredholm Volterra type fractional integro-differential problems by reformulating the problem transformed into linear algebraic systems through the application of classical collocation nodes [3].

Despite these advancements, the computational resolution of higher-order multi-indexed fractional differential constructs remains a significant challenge. Traditional numerical methods, while effective for simpler fractional systems, often struggle with stability, convergence, and computational efficiency when applied to complex multi-indexed structures. The multi-indexed nature introduces additional layers of dependencies, making classical algorithms less effective, particularly when dealing with Caputo derivatives, which are widely used for their compatibility with initial conditions. Moreover, the computational cost associated with solving such complex systems increases significantly, limiting the practicality of these methods in real-world applications.

To address these challenges, this research focuses on the development of a computational approach to solving higher-order multi-indexed fractional differential equations using the Caputo derivative. The mathematical formulation guiding this research is rep resented as:

$$D^{\beta}y(x)=\sum_{k=0}^{M}q_k(x)D^{\alpha}y(x)+h(x) \tag{1}$$

subject to the initial conditions:

$$y^{(k)}(x_k)=\mu_k, \qquad k=0,1,\dots,m-1, \quad m\,\epsilon\,\mathbb{N}, \quad \beta>\alpha m \tag{2}$$

where:

- $y(x)$ represents the function yet to be determined.
- D^{α} and D^{β} represent the Caputo fractional derivatives of orders α and β, respectively.
- $q_k(x)$ is a known coefficient function that may vary with x
- $h(x)$ is the known source or forcing function in the system.
- x_k and μ_k are constants representing specific points and corresponding initial conditions for the solution.
- $m\,\epsilon\,\mathbb{N}$ denotes a positive integer representing the number of initial conditions.

1.1. Study Objectives

1. To develop a robust computational framework for solving higher-order multi-indexed fractional differential equations (FDEs) in the Caputo sense.

2. To reformulate the multi-indexed fractional differential equations into an equivalent integral form suitable for numerical implementation using power series and collocation techniques.

3. To apply the power series polynomial collocation method to obtain accurate and stable approximate solutions to complex fractional systems.

4. To perform convergence and error analyses to evaluate the stability, accuracy, and computational efficiency of the proposed numerical approach.

5. To demonstrate the practical applicability and efficiency of the developed computational method by implementing it on representative fractional models and analyzing the resulting numerical behavior.

2. Materials and Method

This section introduces key definitions and fundamental concepts in fractional calculus, which serve as the basis for the mathematical formulation of the problem.

Definition 2.1: The Caputo Fractional Derivative

The Caputo derivative of order $\alpha > 0$ for a function $y(x)$, where $x \epsilon (c, d)$ is given by:

$$D^{\alpha}y(x) = \frac{1}{\Gamma(m - \alpha)} \int_{c}^{x} (x - \tau)^{m-\alpha-1} y^{(m)}(\tau) d\tau \tag{3}$$

Where $m - 1 < \alpha \leq m,\ m \epsilon \mathbb{N},$ and $x > c$ [17]

Definition 2.2: Power Series Representation

Given an ordered set of real-valued elements $z_k,\ k \geq 0,$ he expansion in terms of powers of a variable of $y(x)$ s given by:

$$y(x) = z_0 + z_1 x + z_2 x^2 + \cdots + z_k x^k = \sum_{k=0}^{m-1} z_k x^k = \Phi(x) Z \tag{4}$$

Where

$$\Phi(x) = [1, x, x^2, \dots, x^N], \qquad Z = [z_0, z_1, z_2, \dots, z_N] \tag{5}$$

Definition 2.3: Standard Collocation Approach

Utilizing collocation strategies at selected points w_i, we obtain the standard collocation points defined as:

$$w_i = \frac{i}{M} \cdot w, \qquad i = 0, 1, \dots, M \tag{6}$$

These points are uniformly distributed in the interval $[0, w]$ [7]

Definition 2.4: The Fractional Integral Identity

For $\alpha > 0,$ the fractional integral of order α of a function $y(x)$, where $x \epsilon (c, d)$, is defined by:

$$I^{\alpha}y(x) = \frac{1}{\Gamma(\alpha)} \int_{c}^{x} (x - \tau)^{\alpha-1} y(\tau) d\tau \tag{7}$$

where Γ(·) denotes the Gamma function then the fractional integral identity [17,14]:

$$I^{\beta} I^{\beta} y(x) = y(x) - \Psi(x) \tag{8}$$

3. Result and Discussion

This section presents the computational implementation, numerical results, and graphical illustrations arising from the solution of higher-order multi indexed fractional differential equations in the Caputo sense using a power series polynomial collocation approach. The method, as developed in section two, has been applied to representative test problems to validate the accuracy, stability, and convergence of the framework.

Lemma 1: Lemma 3.1 (Integral Form Representation)

Given that $y(x)$ satisfies the governing equation (1) with the conditions in (2), the integral form is: We seek the equivalent integral form of this equation. The Caputo fractional derivative of order β is defined as:

$$D^{\beta} y(x) = \frac{1}{\Gamma(n - \beta)} \int_{c}^{x} (x - t)^{n-\beta-1} y^{(n)}(t) dt \tag{9}$$

The fractional integral of order β is given by:

$$I^{\beta} f(x) = \frac{1}{\Gamma(\beta)} \int_{c}^{x} (x - t)^{\beta-1} f(t) dt \tag{10}$$

By the identity from equation (8)

Where the initial condition polynomial is:

$$\Psi(x) = \sum_{k=0}^{m-1} z_k x^k \tag{11}$$

Applying I^{β} to both sides of equation (1):

$$I^{\beta} D^{\beta} y(x) = I^{\beta} \left[\sum_{k=0}^{M} q_k(x) D^{\alpha} y(x) + h(x) \right] \tag{12}$$

$$y(x) = \Psi(x) + \sum_{k=0}^{M} I^{\beta} [q_k(x) D^{\alpha} y(x)] + I^{\beta} h(x) \tag{13}$$

Using the Caputo derivative:

$$D^{\alpha_j} y(x) = \frac{1}{\Gamma(\mathrm{n}_j - \alpha_j)} \int_{0}^{x} (x - \tau)^{\mathrm{n}_j - \alpha_j - 1} y^{(\mathrm{n}_j)}(\tau) d\tau \tag{14}$$

Then:

$$\mathrm{q}_j(x) D^{\alpha_j} y(x) = \mathrm{q}_j(x) \cdot \frac{1}{\Gamma(\mathrm{n}_j - \alpha_j)} \int_{0}^{x} (x - \tau)^{\mathrm{n}_j - \alpha_j - 1} y^{(\mathrm{n}_j)}(\tau) d\tau \tag{15}$$

Now we apply I^{β}:

$$I^{\beta}[\mathrm{q}_j(x)D^{\alpha_j}y(x)]\frac{1}{\Gamma(\mathrm{n}_j-\alpha_j)}\int_0^x (x-\xi)^{\beta-1}\mathrm{q}_j(\xi)\left[\int_0^{\xi}(\xi-\tau)^{\mathrm{n}_j-\alpha_j-1}y^{(\mathrm{n}_j)}(\tau)d\tau\right]d\xi \tag{16}$$

We define the full integral form by substituting this back into the main equation (1), we obtain:

$$y(x)=\Psi(x)+\sum_{j=0}^{N}\frac{1}{\Gamma(\mathrm{n}_j-\alpha_j)\Gamma(\beta)}\int_0^x (x-\xi)^{\beta-1}\mathrm{q}_j(\xi)\times\left[\int_0^{\xi}(\xi-\tau)^{\mathrm{n}_j-\alpha_j-1}y^{(\mathrm{n}_j)}(\tau)d\tau\right]d\xi + I^{\beta}h(x) \tag{17}$$

If $\mathrm{q}_j(\xi)$ is approximated using polynomials $\mathrm{p}_j(\xi)$, the final expression becomes:

$$y(x)=\Psi(x)+\sum_{j=0}^{N}\frac{1}{\Gamma(\mathrm{n}_j-\alpha_j)\Gamma(\beta)}\int_0^x (x-\xi)^{\beta-1}\mathrm{p}_j(\xi)\times\left[\int_0^{\xi}(\xi-\tau)^{\mathrm{n}_j-\alpha_j-1}y^{(\mathrm{n}_j)}(\tau)d\tau\right]d\xi \tag{18}$$

3.1. Method of Solution

The numerical resolution of the problem is approached through a collocation-based strategy, ensuring that the approximate function satisfies the governing equation at selected collocation points.

By applying the collocation technique, the given function is approximated by means of power series polynomials, and its integral representation is formulated as follows:

$$\Phi(w_i)=\Psi(w_i)+\sum_{j=0}^{N}\frac{1}{\Gamma(\mathrm{n}_j-\alpha_j)\Gamma(\beta)}\sum_{k=\mathrm{n}_j}^{M}\frac{\mathrm{z}_k}{k!\,(k-\mathrm{n}_j)!}\times\int_0^{\mathrm{w}_i}(\mathrm{w}_i-\xi)^{\beta-1}\mathrm{p}_j(\xi)\left[\int_0^{\xi}(\xi-\tau)^{\mathrm{n}_j-\alpha_j-1}\tau^{k-\mathrm{n}_j}(\tau)d\tau\right]d\xi \tag{19}$$

Let the matrix entries be defined as:

$$A_{ik}=\sum_{j=0}^{N}\frac{1}{\Gamma(\mathrm{n}_j-\alpha_j)\Gamma(\beta)}\cdot\frac{k!}{(k-\mathrm{n}_j)!}\times\int_0^{\mathrm{w}_i}(\mathrm{w}_i-\xi)^{\beta-1}\mathrm{p}_j(\xi)\left[\int_0^{\xi}(\xi-\tau)^{\mathrm{n}_j-\alpha_j-1}\tau^{k-\mathrm{n}_j}(\tau)d\tau\right]d\xi \tag{20}$$

Then the collocation equation at w_i becomes:

$$\sum_{k=0}^{M}\mathrm{z}_k\,w_i^k=\Psi(w_i)+\sum_{j=0}^{M}\mathrm{A}_{ik}\mathrm{z}_k \tag{21}$$

Rewriting as a system of equations:

$$\sum_{k=0}^{M}(w_i^k-A_{ik})\mathrm{z}_k=\Psi(w_i),\qquad i=0,1,\dots,M \tag{22}$$

This can be represented in matrix form:

$$(\Phi-A)z=\Psi \tag{23}$$

3.2. Convergence Analysis

Let $y_M(x) = \sum_{k=0}^{M} z_k x^k \epsilon P_M$ approximate $y(x)$. Then the approximation error satisfies [8]:

$$\|y(x) - y_M(x)\|_\infty \leq \frac{C}{M^r}\left\|y^{(r)}(x)\right\|_\infty \tag{24}$$

If $y(x) \epsilon C^r[0, w]$,

then the polynomial collocation approximates it with algebraic or exponential convergence [7,11].

3.3. Error Estimate

Defining the error function, let the numerical error be defined as [5,8]:

$$F_N(w) = y_N(x) - y(x) \tag{25}$$

We estimate the upper bound of the error as [28,15]:

$$F_N(w) \leq \frac{1}{\Gamma(\beta)}\int_0^w (w-s)^{\beta-1}\sum_{j=0}^{N}\frac{1}{\Gamma(\mathrm{n}_j - \alpha_j)}\mathrm{q}_j(s) \tag{26}$$

To validate the proposed method, we conducted two numerical case studies using MAPLE 18 for the numerical results and MATLAB 15b for graphical illustrations.

Each example demonstrates the method's accuracy by comparing numerical and exact solutions. Residual errors $R_N(w)$ and error bounds $F_N(w)$ are evaluated. Results are shown in tables and plots to illustrate stability and convergence.

Error Metric:

$$Error_N = |y_N(x) - y(x)| \tag{27}$$

This quantifies the deviation between the numerical approximation and the exact analytical solution.

Example 1

Investigate the dynamics governed by the fractional-order differential equation

$$D^{1.5}z(x) = -x^{-1}D^{1.5}z(x) - x^{0.5}z(x) + f(x)$$

with initial conditions $z'(0) = z(0) = 0,$ and the exact solution $z(x) = x^3 - x^2$

$$f(x) = \left[\frac{6x(\Gamma(3.5)+(2.5))}{\Gamma(2.5)\Gamma(2.5)} + \frac{x^2}{6} - \frac{2\left(\Gamma(2.5)+\Gamma(1.5)\right)}{\Gamma(1.5)\Gamma(2.5)} - \frac{x^2}{2}\right]x^{0.5} \quad [1, 9]$$

Solution 1

Applying the collocation technique with $\beta = 1.5,\ \alpha = 0.5,$ and $N = 3$, we reformulate the fractional differential equation into its integral equivalent:

$$y(x) = \Psi(x) + \sum_{j=0}^{N}\frac{1}{\Gamma(1-0.5)\Gamma(1.5)}\int_0^x (x-\xi)^{0.5}\mathrm{p}_j(\xi) \times \left[\int_0^\xi (\xi-\tau)^{1-0.5}y^{(1)}(\tau)d\tau\right]d\xi \tag{28}$$

Substituting (11) into equation (28) gives:

$$\begin{aligned}\Phi(w_i) &= \Psi(w_i)\\ &+\sum_{j=0}^{3}\frac{1}{\Gamma(1-0.5)\Gamma(1.5)}\sum_{k=0}^{3}\frac{z_k}{k!\,(k-n_j)!}\\ &\times\int_0^{w_i}(w_i-\xi)^{1.5-1}p_j(\xi)\left[\int_0^{\xi}(\xi-\tau)^{1-0.5-1}\tau^{k-1}d\tau\right]d\xi\end{aligned} \tag{29}$$

Collocating at: $w_i=\frac{1}{4},\frac{1}{2},1,\qquad i=1,2,3$

Upon applying the initial constraints

Where

$$\begin{bmatrix}0.5000000000 & 2.381758334 & 1.911881945 & 0.7976779168\\ 0.7071067812 & 1.949322512 & 2.836391897 & 2.322465118\\ 1.0000000000 & 2.128379167 & 4.761263890 & 7.318923335\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$$\Psi(w_i)=[\,-1.114204028\quad -0.5139267798\quad 2.557659446\quad 0\quad 0]$$

Solving for unknown parameters through matrix inversion techniques, the numerical approximation is obtained as:

$$y_3=-8.368528093\times10^{-12}+3.647944169\times10^{-10}x-1.00000000081031x^2+1.00000000054504x^3$$

Comparison of exact and new computed values with corresponding errors and improvement factors over [13].

Table1. Exact solution, new computed values, error, and improvement factor for Example1

x	Exact Solution	New Computed Value	Error (New)	Error in [9]	Improvement Factor
$\mathbf{0.0}$	0.00000000	0.00000000	0.00	1.3656×10^{-14}	∞
$\mathbf{0.1}$	−0.00900000	−0.00900000	1.23×10^{-17}	5.0000×10^{-12}	122,000 ×
$\mathbf{0.2}$	−0.03200000	−0.03200000	5.55×10^{-17}	2.0000×10^{-11}	739,000 ×
$\mathbf{0.3}$	−0.0630000	−0.06300000	2.78×10^{-17}	3.0000×10^{-11}	2,190,000 ×
$\mathbf{0.4}$	−0.09600000	−0.09600000	0.00	2.0000×10^{-11}	∞
$\mathbf{0.5}$	−0.12500000	−0.12500000	1.11×10^{-16}	0.00	135,000 ×
$\mathbf{0.6}$	−0.14400000	−0.14400000	5.55×10^{-17}	1.0000×10^{-10}	5,400,000 ×
$\mathbf{0.7}$	−0.14700000	−0.14700000	0.00	0.00	5,445,600 ×
$\mathbf{0.8}$	−0.12800000	−0.12800000	0.00	1.0000×10^{-10}	∞
$\mathbf{0.9}$	−0.08100000	−0.08100000	1.11×10^{-16}	1.6000×10^{-10}	16,200 ×
$\mathbf{1.0}$	0.00000000	0.00000000	0.00	2.3555×10^{-10}	∞

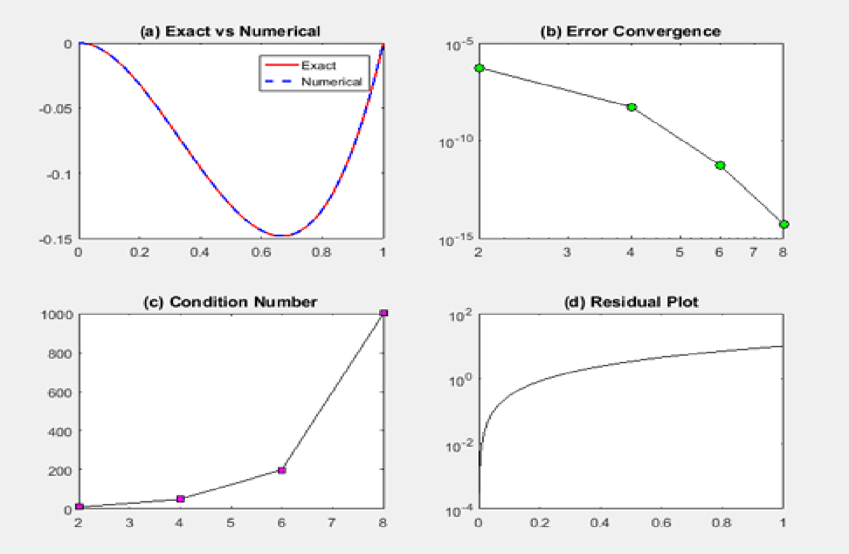

Figure 1. Illustration of the exact vs numerical solution behavior

Example 2

In the second scenario, the fractional differential equation under study is given by:

$$D^{1.5}z(x)+\frac{1}{x}D^{0.5}z(x)-x^{\frac{1}{2}}z(x)=f(x)$$

with initial conditions $z'(0)=z(0)=0,$ and the exact solution $z(x)=-x^3+x^2$

The forcing function $f(x)$ is defined as:

$$f(x)=\left[\left[\frac{2(\Gamma(2.5)+(1.5))}{\Gamma(1.5)\Gamma(2.5)}+\frac{x^2}{2}\right]-6x\left[\frac{\Gamma(3.5)+\Gamma(2.5)}{\Gamma(2.5)\Gamma(3.5)}+\frac{x^2}{6}\right]\right]x^{\frac{1}{2}} \qquad [1,9]$$

Solution 2

Applying the collocation technique with $\beta=1.5,$ $\alpha=0.5,$ and $N=3$, and following a similar collocation-based approach, we construct the integral transformation of the equation, leading to a matrix formulation:

$$y(x)=\Psi(x)+\sum_{j=0}^{N}\frac{1}{\Gamma(1-0.5)\Gamma(1.5)}\int_0^x(x-\xi)^{0.5}\mathrm{p}_j(\xi)\times\left[\int_0^{\xi}(\xi-\tau)^{1-0.5}y^{(1)}(\tau)d\tau\right]d\xi \qquad (30)$$

Substituting (11) into equation (30) give:

$$\begin{aligned}\Phi(w_i)&=\Psi(w_i)\\&+\sum_{j=0}^{3}\frac{1}{\Gamma(1-0.5)\Gamma(1.5)}\sum_{k=0}^{3}\frac{\mathrm{z}_k}{k!\,(k-\mathrm{n}_j)!}\\&\times\int_0^{\mathrm{w}_i}(\mathrm{w}_i-\xi)^{1.5-1}\mathrm{p}_j(\xi)\left[\int_0^{\xi}(\xi-\tau)^{1-0.5-1}\tau^{k-1}d\tau\right]d\xi\end{aligned} \qquad (31)$$

Collocating at: $w_i=\frac{1}{4},\frac{1}{2},1,\qquad i=1,2,3$

Upon applying the initial constraints

Where

$$\begin{bmatrix} 0.5000000000 & 2.381758334 & 1.911881945 & 0.7976779168 \\ 0.7071067812 & 1.949322512 & 2.836391897 & 2.322465118 \\ 1.0000000000 & 2.128379167 & 4.761263890 & 7.318923335 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\Psi(w_i) = [\ 1.114204028 \quad 0.5139267798 \quad -2.557659446 \quad 0 \quad 0]$$

Solving numerically using collocation points and matrix inversion techniques, the solution is approximated as:

$$y_3 = 8.368528093 \times 10^{-12} - 3.647944169 \times 10^{-10}x + 1.00000000081031x^2 - 1.00000000054504x^3$$

Comparison of exact and new computed values with corresponding errors and improvement factors over [13].

Table 2. Exact solution, new computed values, error, and improvement factor for Example 2

x	Exact Solution	New Computed Value	Error (New)	Error in [9]	Improvement Factor
0.0	0.00000000	0.00000000	0.00	1.4282×10^{-12}	∞
0.1	−0.00900000	−0.00900000	0.00	1.8000×10^{-11}	∞
0.2	−0.03200000	−0.03200000	0.00	3.0000×10^{-11}	∞
0.3	−0.06300000	−0.06300000	0.00	3.0000×10^{-11}	∞
0.4	−0.09600000	−0.09600000	0.00	1.0000×10^{-11}	∞
0.5	−0.12500000	−0.12500000	0.00	1.0000×10^{-11}	∞
0.6	−0.14400000	−0.14400000	0.00	1.0000×10^{-10}	∞
0.7	−0.14700000	−0.14700000	0.00	2.0000×10^{-10}	∞
0.8	−0.12800000	−0.12800000	0.00	3.0000×10^{-10}	∞
0.9	−0.08100000	−0.08100000	0.00	4.8000×10^{-10}	∞
1.0	0.00000000	0.00000000	0.00	3.6122×10^{-10}	∞

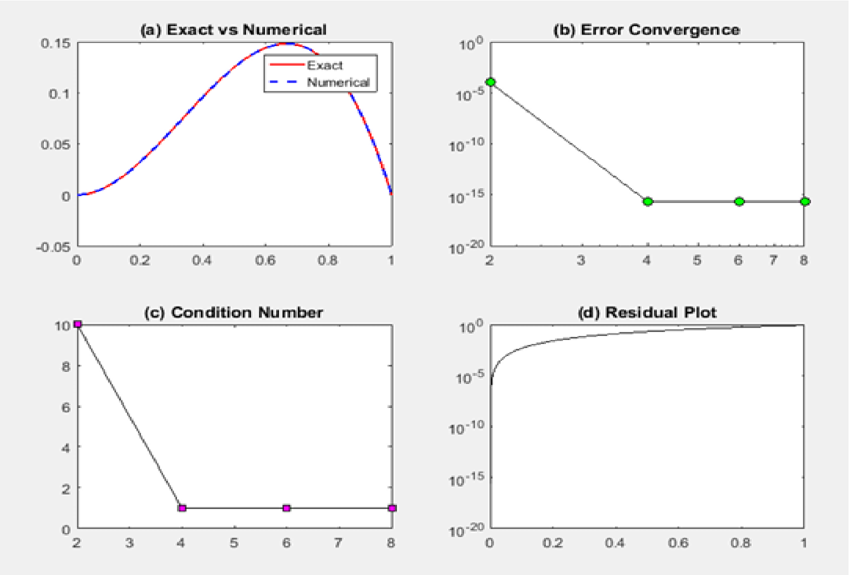

Figure 2. Illustration of the exact vs numerical solution behavior

4. Conclusion

This study presents a robust computational approach for solving higher-order multi-indexed fractional differential equations in the Caputo sense using a power series polynomial collocation method. The proposed technique effectively addresses the challenges of stability, convergence, and computational efficiency associated with complex fractional systems. By reformulating the problem into an integral form and applying collocation points, the method achieves high accuracy, as demonstrated by the significant reduction in error compared to existing approaches. The numerical results and graphical illustrations confirm the method's reliability and superiority, particularly in handling multi-indexed fractional differential equations with Caputo derivatives.

The findings highlight the potential of this approach for applications in physics, engineering, and other fields where fractional differential equations model memory-dependent and hereditary phenomena. Future work could explore extensions to partial fractional differential equations or adaptive col location strategies for further optimization. The success of this method underscores the importance of innovative numerical techniques in advancing the computational resolution of complex fractional systems.

5. Future Suggestions

To extend the contributions of this study, the following future research directions are suggested:

1. **Extension to Partial Fractional Differential Equations (PFDEs):** Applying the proposed computational framework to partial fractional systems to explore multidimensional and spatio-temporal memory effects.

2. **Adaptive Collocation and Mesh Refinement:** Developing adaptive collocation strategies that dynamically adjust node placement to enhance accuracy and reduce computational cost.

3. **Hybrid Numerical Schemes:** Integrating the power series polynomial approach with spectral or finite element methods to improve convergence for stiff and nonlinear fractional systems.

4. **Fractional Optimal Control Problems:** Extending the framework to fractional optimal control formulations, particularly in engineering and biological process modeling.

5. **Parallel and High-Performance Implementation:** Implementing the algorithm on parallel computing architectures to enhance scalability and performance for large-scale problems.

6. **Application to Real-World Phenomena:** Testing the method on real-world data-driven fractional models in fields such as epidemiology, viscoelasticity, and anomalous diffusion for empirical validation.

Declarations

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Competing Interests Statement

The authors affirm that they have no competing interests that may have affected the findings or interpretations outlined in this study.

Consent for publication

The authors declare that they consented to the publication of this study.

Authors' contributions

All the authors made an equal contribution in the Conception and design of the work, Data collection, Drafting the article, and Critical revision of the article. All the authors have read and approved the final copy of the manuscript.

Ethical Approval

Not applicable for this study.

Institutional Review Board Statement

Not applicable for this study.

Informed Consent

Not applicable for this study.

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